each arc of 15°. Is this striking disparity merely accidental? or has it resulted from the operation of a physical cause?

The fact may perhaps be sufficiently explained by the remark of Prof. Newcomb that "there is always a tendency in the perihelia of the asteroids to coincide in longitude with the perihelion of Jupiter, and in their nodes to coincide in longitude with the node of Jupiter."

THE RECURRENCE OF ECLIPSES.

BY PROF. DAVID TROWBRIDGE, WATERBURGH, N. Y.

That eclipses recur in the same order in a cycle of about eighteen years was known to the ancient Chaldeans, who probably discovered the period from observation, by comparing together the records of many eclipses. This period, which they called the *saros*, must have been of great advantage to the ancient astronomer in predicting eclipses; since a record of all the solar eclipses (on an ayerage about 41), and of all the lunar eclipses (on an average about 29) in the order in which they occurred, during any one of the complete cycles, would enable him to predict approximately the eclipses of the next succeeding period or cycle. The coincidences required, however, are not sufficiently exact to give more than approximate results; and if there are several intervening cycles, the recurrence is not very reliable even as an approximation. I have never seen in any astronomical work any other periods referred to, (though it is quite possible that *some* work may contain such reference), though other and much more exact periods exist, and one of them only about three times the length of the *saros*, as I shall now show.

According to Bessel the length of the siderial year is 365.2563582 mean solar days.

The mean sidereal revolution of the moon's nodes is equal to 6793.39108 mean solar days.

The revolution of the moon's nodes being accomplished in a direction opposite to the apparent revolution of the sun, if they set out together at any time, they will again come together in less than a year; or really in 346.619848 mean solar days. This is called the mean (as all these periods are mean periods) synodical revolution of the moon's nodes. The mean synodical revolution of the moon, or the period from one new moon to the next succeding new moon, is 29.5305887 mean solar days. The several approximate ratios of these last two numbers will make known to us the

time required for the sun, moon and nodes, all setting out from the same point in the heavens, to return, approximately, to the same relative mean position. If we reduce the ratio of these periods to a continued fraction, we shall have

$$\frac{346.6198480}{29.5305887}=11+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{5+\cfrac{1}{47+}}}}}}}$$

The several approximate ratios, or the sums of the partial fractions, are

$$\tfrac{11}{1},\ \tfrac{12}{1},\ \tfrac{35}{3},\ \tfrac{47}{4},\ \tfrac{223}{19},\ \tfrac{716}{61},\ \tfrac{3803}{324},\ \tfrac{179457}{15289},\ \&c.$$

The fifth ratio in this series is that known as the *saros*. The sixth and seventh, so far as I know, have not been given before, and the eighth one is too long to be useful. The errors of the periods are as follows:

$$\begin{array}{lll} 29^{d}.5305887 \times 223 & = 6585^{d}.32128 & = 18^{y} \text{ (of 365 days) and } 15^{d}.231. \\ 346^{d}.619848 \ \times 19 & = 6585^{d}.77711. & \\ \quad\text{Difference,} & \quad 0^{d}.45583 & = 10^{h}.94. \end{array}$$

$$\begin{array}{lll} 29^{d}.5305887 \times 716 & = 21143^{d}.901509 & = 57^{y} \text{ (of 365 days) and } 338^{d}.901. \\ 346^{d}.619848 \ \times 61 & = 21143^{d}.810728. & \\ \quad\text{Difference,} & \quad 0^{d}.090781 & = 2^{h}.1787. \end{array}$$

$$\begin{array}{lll} 29^{d}.5305887 \times 3803 & = 112304^{d}.828826 & = 307^{y} \text{ (of 365 days) and } 249^{d}.828. \\ 346^{d}.619848 \ \times 324 & = 112304^{d}.830752. & \\ \quad\text{Difference,} & \quad 0^{d}.001926 & = 0^{h}2^{m}.773. \end{array}$$

Although these numbers are the mean values, yet the true values will only change the character of the eclipse, and not prevent it from taking place. This is especially true in the periods of 57 and 307 years. Ten successive recurrences of the 307-year cycle will not be half an hour from exact coincidence. It seems, therefore, that this period will serve as a check in computing the time when ancient eclipses happened.

A total eclipse of the sun is referred to by Herodotus, which has been the subject of much discussion. Baily placed it by his calculations on the 30th of September, 610 B. C. Eight times the 307-year cycle brings us to the 28th of July, 1851, the great total eclipse of that year.

Prof. Airy's calculations fixed the time of the eclipse on the 28th of May,

585 B. C. Eight times the 307-year cycle brings us to April 24, 1876. It does not appear that any eclipse will happen at that time.

Another eclipse that seems to be referred to by Xenophon happened, according to Prof. Airy, May 19, 556 B. C. Eight 307-year periods, minus one 57-year period, brings us to April 27, 1847. No eclipse happened at that time.

The eclipse of March 20, 1140 A. D., total at London, returned December 21, 1870, after two 307-year periods, plus two 57-year periods. The eclipse of August 21, 1560, returned February 22, 1868, after one 307-year period; and the eclipse of April 9, 1567, returns October 9, 1874, after one 307-year period.

Of instances of the 57-year period we may mention that the eclipse of July 14, 1748, returned in 1806, June 16th; and this last returned May 5, 1864. The great eclipse of June 24, 1778, returned in 1836, May 15th, as an annular eclipse, after one period.

So far as I have been able to compare these periods with eclipses whose dates are certain, they give the order of the eclipses.

OPERATIONS ON IMAGINARY QUANTITIES CONSIDERED GEOMETRICALLY.

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In the view here taken of so-called imaginary quantities the $\sqrt{-1}$ is not considered as a quantity at all, but merely as a sign of an operation. First, let us consider the multiplication of binomial expressions. The following table of signs shows that there are sixteen cases:—

+ +	+ +	+ +	+ +
+ +	+ —	— +	— —
+ —	+ —	+ —	+ —
+ +	+ —	— +	— —
— +	— +	— +	— +
+ +	+ —	— +	— —
— —	— —	— —	— —
+ +	+ —	— +	— —